

QP Code

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Register Number

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VELALAR COLLEGE OF ENGINEERING AND TECHNOLOGY

(An Autonomous Institution, Affiliated to Anna University, Chennai)

Semester Examinations – Apr / May 2017

Regulations-2016

Programme: B.E/B.Tech

Semester: 1

Max. Marks: 100

Duration 3 Hrs

Course Code & Title: **16MAT11 MATHEMATICS-I**

Knowledge
Levels (KL)

K1 - Remembering
K2 - Understanding

K3 - Applying
K4 - Analyzing

K5 - Evaluating
K6 - Creating

Part A - Answer ALL Questions.

10 x 2 = 20 Marks

- | No. | Question | KL |
|-----|---|----|
| 1. | If -1, 2, 3 are the eigen values of A, then find the eigen values of A^2 , and A^{-1} . | K4 |
| 2. | Show that the matrix $P = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ is orthogonal. | K3 |
| 3. | Show that the series $1 + \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} + \dots$ is absolutely convergent. | K3 |
| 4. | State the D'Alembert's ratio test for Convergence of a positive term series. | K1 |
| 5. | Find the radius of curvature for $y = e^x$ at the point where it cuts the y axis. | K3 |
| 6. | Find the envelope of the family of straight lines $y = mx + \frac{a}{m}$ where m is the parameter. | K5 |
| 7. | If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}$ | K5 |
| 8. | If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ | K3 |
| 9. | Find the area of a circle of radius 'a' with using double integration. | K5 |
| 10. | Evaluate $\int_0^1 \int_1^2 x(x+y) dy dx$ | K5 |

Part B - Answer ALL Questions.

5 x 16 = 80 Marks

- | No | Question | Marks | KL |
|-----|---|-------|----|
| 11. | a Reduce the following quadratic forms into a canonical form by an orthogonal reduction and find the rank, signature, index and the nature of the quadratic form. | 16 | K3 |

$$6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$$

OR

- | | | | |
|---|--|----|----|
| b | Diagonalise the matrix $A = \begin{bmatrix} 6 & -2 & -2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ using orthogonal transformation. | 16 | K3 |
|---|--|----|----|

12. a i. Test the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{\cos n\pi}{n^2 + 1} \right)$. 8 K6
- ii. Test for conditional convergence of the following series 8 K4
- $$\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \dots$$
- OR
- b i. Use Integral test to discuss the nature of convergence of the series 8 K4
- $$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots$$
- ii. Test the convergence of the series 8 K4
- $$\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \dots, 0 < x < 1$$
13. a i. Find the radius of curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2} \right)$ on the curve 8 K4
- $$x^3 + y^3 = 3axy.$$
- ii. Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 8 K4
- OR
- b i. Find the equation of circle of curvature at (c,c) on $xy=c^2$ 8 K3
- ii. Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ where a and b are connected by the relations $a+b=c$, 8 K3
14. a i. Find the Taylor's series expansion of $e^x \sin y$ near the point $\left(-1, \frac{\pi}{4} \right)$ upto 8 K3
- the third degree terms.
- ii. A rectangular box open at the top, is to have volume of 32 cc. Find the 8 K3
- dimensions of the box, that requires the least material for its construction
- OR
- b i. If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x - y} \right)$ Prove that $x \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \sin 2u$. 8 K3
- ii. Find the maximum value of the largest rectangular parallelepiped that can 8 K3
- be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
15. a i. Change the order of integration and then evaluate $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy dy dx$. 8 K3,K
- 6
- ii. Find the area between the parabola $y = x^2$ and the line $y = x$. 8 K3
- OR
- b i. Transform into polar coordinates and evaluate 8 K3,K
- $$\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dy dx$$
- 6
- ii. Find the area of the cardioid $r = a(1 + \cos \theta)$ 8 K3